Ex. 1)
For the T-beam section shown below, calculate the cracking torque and check the section adequacy for shear and torsion. Consider the factored shear force and the factored torsion at the critical section to be $V_u = 150 \text{kN}$ and $T_u = 45 \text{kNm}$. Use $d = 690 \text{mm}$, $f'c = 28 \text{MPa}$, and $f_y = 420 \text{MPa}$.

\[ \text{Sol.)} \]
1- Cracking torque

\[ A_{cp} = 300 \times (750 - 150) + 700 \times 150 = 285000 \text{ mm}^2 \]

\[ P_{cp} = 700 + 2(150) + (700 - 300) + 2(750 - 150) + 300 = 2900 \text{ mm} \]

\[ \phi T_{cr} = \phi 0.33 \lambda \sqrt{f'_c f_{cp}} = 0.75 \times 0.33(1.0)\sqrt{28} \left(\frac{285000} {2900}\right)^2 \times 10^{-6} = 36.7 \text{ kN.m} \]

2- Check the section adequacy for torsion and shear
Assume 40 mm cover and 10 mm bars for torsion and shear reinforcement.

Section properties of web:

\[ x_o = 300 - 2 \left(40 + \frac{10}{2}\right) = 210 \text{ mm} \]

\[ y_o = 750 - 2 \left(40 + \frac{10}{2}\right) = 660 \text{ mm} \]

Section properties of flange:

\[ x_o = 700 - 2 \left(40 + \frac{10}{2}\right) - 210 = 610 - 210 = 400 \text{ mm} \]
\[ y_o = 150 - 2 \left( 40 + \frac{10}{2} \right) = 60 \text{ mm} \]

\[ A_{oh} = 210 \times 660 + 400 \times 60 = 162600 \text{ mm}^2 \]

\[ P_h = 610 + 2(60) + 400 + 2(660 - 60) + 210 = 2540 \text{ mm} \]

\[ \frac{V_u}{b_w d} + \left( \frac{T_u P_h}{1.7 A_{oh}^2} \right)^2 \leq \phi \left( \frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) \]

\[ V_c = 0.17 \lambda \sqrt{f'_c b_w d} = 0.17(1.0) \sqrt{28} \times 300 \times 690 \times 10^{-3} = 186.2 \text{ kN} \]

\[ \frac{V_u}{b_w d} + \left( \frac{T_u P_h}{1.7 A_{oh}^2} \right)^2 = \sqrt{\left( \frac{150 \times 10^3}{300 \times 690} \right)^2 + \left( \frac{45 \times 10^6 \times 2540}{1.7(162600)^2} \right)^2} = 2.64 \text{ MPa} \]

\[ \phi \left( \frac{V_c}{b_w d} + 0.66 \sqrt{f'_c} \right) = 0.75 \left( \frac{186.2 \times 10^3}{300 \times 690} + 0.66 \sqrt{28} \right) = 3.29 \text{ MPa} > 2.64 \text{ MPa} \]

\[ \therefore \text{ The section is adequate} \]

**Ex. 2)**

For the T-beam section shown below, calculate the cracking torque and check the section adequacy for shear and torsion. Consider the factored shear force and the factored torsion at the critical section to be \( V_u = 150 \text{ kN} \) and \( T_u = 45 \text{ kN.m} \). Use \( d = 690 \text{ mm} \), \( f'_c = 28 \text{ MPa} \), and \( f_y = 420 \text{ MPa} \).
Sol.)

Since only the web is enclosed by closed stirrups, then consider only the web dimensions for torsion calculations. Thus, consider the T-beam as a rectangular beam with $b = 300$ mm and $h = 750$ mm.

1- Cracking torque

$A_{cp} = 300 \times 750 = 225000$ $mm^2$

$P_{cp} = 2(300 + 750) = 2100$ mm

$\phi T_{cr} = \phi 0.33 \lambda \sqrt{f'_{c}} \frac{A_{cp}}{P_{cp}} = 0.75 \times 0.33(1.0)\sqrt{28 \frac{(225000)^2}{2100}} \times 10^{-6} = 31.6$ $kN.m$

2- Check the section adequacy for torsion and shear

Assume 40 mm cover and 10 mm bars for torsion and shear reinforcement.

$x_o = 300 - 2\left(40 + \frac{10}{2}\right) = 210$ mm

$y_o = 750 - 2\left(40 + \frac{10}{2}\right) = 660$ mm

$A_{oh} = 210 \times 660 = 138600$ $mm^2$

$P_h = 2(210 + 660) = 1740$ mm

$\sqrt{\left(\frac{V_u}{b_{wd}}\right)^2 + \left(\frac{T_u P_h}{1.7 A_{oh}^2}\right)^2} \leq \phi \left(\frac{V_c}{b_{wd}} + 0.66\sqrt{f'_{c}}\right)$

$V_c = 0.17 \lambda \sqrt{f'_{c}} b_{wd} = 0.17(1.0)\sqrt{28 \times 300 \times 690 \times 10^{-3}} = 186.2$ $kN$

$\sqrt{\left(\frac{V_u}{b_{wd}}\right)^2 + \left(\frac{T_u P_h}{1.7 A_{oh}^2}\right)^2} = \sqrt{\left(\frac{150 \times 10^3}{300 \times 690}\right)^2 + \left(\frac{45 \times 10^6 \times 1740}{1.7(138600)^2}\right)^2} = 2.5$ $MPa$

$\phi \left(\frac{V_c}{b_{wd}} + 0.66\sqrt{f'_{c}}\right) = 0.75 \left(\frac{186.2 \times 10^3}{300 \times 690} + 0.66\sqrt{28}\right) = 3.29$ $MPa > 2.5$ $MPa$

$\therefore$ The section is adequate

Ex. 3)

The T-beam shown below has a span of 8.5 $m$ and is carrying a total factored load of 20 $kN/m$.

If the supports prevent the rotation of the member in planes perpendicular to its longitudinal axis, then determine the eccentricity ($e$) of the load at which no need to add reinforcement to resist torsion. $f'_{c} = 25$ $MPa$ , $d = 520$ mm
Sol.)

$P_u, \text{critical}$ is at distance $d$ from support.

\[
\therefore P_u d = w_u \left(\frac{8.5}{2} - 0.52\right) = 20(4.25 - 0.52) = 74.6 \text{ kN}
\]

\[T_u = P_u \cdot e = 74.6 \times e \text{ kN} \cdot \text{mm}\]

Calculate minimum requirement for torsion reinforcement

If \[T_u \leq \emptyset 0.083 \lambda \sqrt{f'c} \frac{A_{cp}^2}{P_{cp}}\] then, no need for torsion reinforcement. Thus, use:

\[T_u = \emptyset 0.083 \sqrt{f'c} \frac{A_{cp}^2}{P_{cp}}\]

\[A_{cp} = 480 \times 300 + 120 \times 1000 = 264000 \text{ mm}^2\]

\[P_{cp} = 300 + 2 \times 480 + 2 \times 350 + 2 \times 120 + 1000 = 3200 \text{ mm}\]

\[\therefore 74.6 \times 10^3 \times e = 0.75 \times 0.083(1.000)\sqrt{25} \frac{(264000)^2}{3200} = 6779025\]

\[\therefore e = \frac{6779025}{74.6 \times 10^3} = 90.9 \text{ mm}\]

Ex. 4)

A rectangular beam with section width of 350 mm and section depth of 630 mm is subjected to an external factored shear force acts at the critical section with a value of $V_u=200 \text{ kN}$ and an equilibrium factored external torsional moment of $T_u= 50 \text{ kN} \cdot \text{m}$. If the required bending reinforcement is $A_s = 1500 \text{ mm}^2$, $f'c = 28 \text{ MPa}$, and $f_y = 420 \text{ MPa}$, then check the adequacy of the section and calculate the required torsion and shear reinforcement.
Sol.)

1- Check whether torsion effect can be neglected or not.

\[
A_{cp} = 350 \times 630 = 220500 \text{ mm}^2
\]

\[
P_{cp} = 2(x + y) = 2(350 + 630) = 1960 \text{ mm}
\]

\[
\phi T_{th} = \phi 0.083 \lambda \sqrt{f_c} \frac{A_{cp}^2}{P_{cp}} = 0.75 \times 0.083(1.0)\sqrt{28} \frac{(220500)^2}{1960} \times 10^{-6} = 8.17 \text{ kN.m}
\]

\[
T_u = 50 \text{ kN.m} > \phi T_{th} = 8.17 \text{ kN.m}
\]

\[\therefore \text{Torsion must be considered} \]

Section properties: assume 40 mm cover, 12 mm diameter stirrups, and 20 mm flexural bars.

\[
x_0 = 350 - 2 \left(40 + \frac{12}{2}\right) = 258 \text{ mm}
\]

\[
y_0 = 630 - 2 \left(40 + \frac{12}{2}\right) = 538 \text{ mm}
\]

\[
A_{oh} = x_0 y_0 = 258 \times 538 = 138804 \text{ mm}^2
\]

\[
P_h = 2(x_0 + y_0) = 2(258 + 538) = 1592 \text{ mm}
\]

\[
A_o = 0.85 A_{oh} = 117983 \text{ mm}^2
\]

\[
d = 630 - \left(40 + 12 + \frac{20}{2}\right) = 568
\]

use \( \theta = 45^\circ \Rightarrow \cot \theta = 1.0 \)

2- Check the adequacy of section for combined shear and torsion.

\[
\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7 A_{oh}}\right)^2} \leq \phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f_c'}\right)
\]

\[
V_c = 0.17 \lambda \sqrt{f_c'} b_w d = 0.17(1.0) \sqrt{28} \times 350 \times 568 \times 10^{-3} = 178.83 \text{ kN}
\]

\[
\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u P_h}{1.7 A_{oh}}\right)^2} = \sqrt{\left(\frac{200 \times 10^3}{350 \times 568}\right)^2 + \left(\frac{50 \times 10^6 \times 1592}{1.7(138804)^2}\right)^2} = 2.63 \text{ MPa}
\]

\[
\phi \left(\frac{V_c}{b_w d} + 0.66 \sqrt{f_c'}\right) = 0.75 \left(\frac{178.83 \times 10^3}{350 \times 568} + 0.66 \sqrt{28}\right) = 3.29 \text{ MPa} > 2.63 \text{ MPa}
\]

\[\therefore \text{The section is adequate} \]

3- Torsional reinforcement

\[
T_n = \frac{T_u}{\phi} = \frac{50}{0.75} = 66.66 \text{ kN.m}
\]
4- Shear reinforcement

\[ \phi V_c = 0.75 \times 178.83 = 134.12 \text{kN} \]

\[ V_u = 200 > \phi V_c = 134.12 \text{kN} \quad \therefore \text{provide stirrups} \]

\[ V_s = \frac{V_u - \phi V_c}{\phi} = \frac{(200 - 134.12)}{0.75} = 87.84 \text{ kN} \]

\[ A_v = \frac{V_s}{f_y d} = \frac{87.84 \times 10^3}{420 \times 568} = 0.368 \text{ mm}^2/\text{mm}/2 \text{ legs} \]

5- Total stirrups reinforcement

\[ \frac{A_v + A_t}{s} = \frac{2A_t}{s} + \frac{A_v}{s} = 2 \times 0.673 + 0.368 = 1.714 \text{ mm}^2/\text{mm}/2 \text{ legs} \]

Using \( \phi 12 \text{ mm closed stirrups: Area of two legs} = 2 \times 113 = 226 \text{ mm}^2 \)

\[ \frac{226}{s} = 1.714 \quad \text{Thus} \quad s = \frac{226}{1.714} = 131 \text{ mm} \]

Check maximum spacing:

a- Maximum spacing for torsion

\[ s_{max} = \left\{ \begin{array}{l}
\frac{ph}{8} = \frac{1592}{8} = 199 \text{ mm (Controls)} \\
300 \text{ mm}
\end{array} \right. \]

b- Maximum spacing for shear

\[ V_s = 87.84 \text{ kN} < \phi 0.33 \sqrt{f'_c b_w d} = 260.3 \]

\[ s_{max} = \left\{ \begin{array}{l}
\frac{d}{2} = \frac{568}{2} = 284 \text{ mm} \\
600 \text{ mm}
\end{array} \right. \]

\[ : S = 131 \text{ mm} < S_{max} = 199 \text{ mm} \]

\[ : \text{use} \ \phi 12 \text{ mm closed stirrups at 130 mm} \]

6- Calculate longitudinal torsion reinforcement

\[ A_t = \frac{A_t}{s} P_h \frac{f_{yt}}{f_y} \cot^2 \theta = 0.673 \times 1592 \times 1.0 \times 1.0 = 1071 \text{ mm}^2 \]

Check minimum longitudinal reinforcement:

\[ (a) \ A_{t\ min} = 0.42 \sqrt{f'_c} \frac{A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) P_h \frac{f_{yt}}{f_y} \]
\[ A_{t\,min} = 0.42 \sqrt{f'c'} \frac{A_{cp}}{f_y} \left( \frac{0.175 b_w}{f_y} \right) p_h \frac{f_y t}{f_y} \]

\[ = 0.42 \sqrt{28} \times \frac{220500}{420} \left( \frac{0.175 \times 350}{420} \right) \times 1592 \times 1.0 = 934 \, mm^2 \]

\[ \therefore A_t = 1071 \, mm^2 > A_{t\,min} = 95 \, mm^2 \]

Since the beam effective depth is less than 600 mm and the maximum vertical spacing between longitudinal bars is 300 mm, then one bar is required at each face at the mid depth of the section. Then, \( \frac{1}{3} A_t \) goes to the top corners and \( \frac{1}{3} A_t \) goes to the bottom corners to be added to the flexural bars, while the remaining \( \frac{1}{3} A_t \), would thus be distributed at the mid depth of the beam section.

\[ \sum A_s = A_t \frac{1}{3} + A_s = \frac{1071}{3} + 1500 = 1857 \, mm^2 \]

\[ N = \frac{1857}{314} = 5.9 \]

Thus, Use 6Ø20 at the bottom of the beam

\[ \frac{A_t}{3} = \frac{1071}{3} = 357 \, mm^2 \]

Thus, Provide 2Ø16 at the top corners, \( A_s = 402 \, mm^2 \)

And provide 2Ø16 at mid depth of the beam section, \( A_s = 402 \, mm^2 \)

\[ \begin{array}{c}
\text{Ø12@130 mm closed stirrups} \\
2016 \\
2016 \\
6020 \\
630 \, mm \\
350 \, mm \\
\end{array} \]

**Ex. 5)**

Design the beam in Example 2, considering the following two loading cases.

a) Compatibility factored torque \( T_u = 8 \, kN.m \)

b) Compatibility factored torque \( T_u = 40 \, kN.m \)
Sol.)

a) Compatibility torsion, \( T_u = 8.0 \text{kN.m} \)

Check if torsion can be neglected:
\[ T_u = 8.0 < \phi T_{th} = 8.17 \text{kN.m} \]
\[ \therefore \text{Neglect torsion effect and design for shear only.} \]

From Example 2, \( \frac{A_v}{s} = 0.368 \text{ mm}^2/\text{mm}/2 \text{ lags} \)

\[ s = \frac{2 \times 113}{0.368} = 614 \text{ mm} \]

\[ s_{max} = \left( \frac{d}{2} \right) = \frac{568}{2} = 284 \text{ mm} \]

\[ \therefore \text{Use } \phi 12 \text{ mm stirrups at 280 mm.} \]

b) Compatibility torsion \( T_u = 40 \text{kN.m} \)

Check if torsion can be neglected:
\[ T_u = 40 > \phi T_{th} = 8.17 \text{kN.m} \]
\[ \therefore \text{Torsion must be considered.} \]

\[ \phi T_{cr} = \phi 0.33 \sqrt{\frac{f'c}{f_{cy}}} \frac{A_{cp}^2}{P_{cp}} = 0.75 \times 0.33(1.0)\sqrt{\frac{220500}{1960}} \times 10^{-6} = 32.5 \text{kN.m} \]

\[ T_u = 40 > \phi T_{cr} = 32.5 \text{kN.m} \]

Thus, design torque can be reduced to \( T_u = \phi T_{cr} = 32.5 \text{kN.m} \)

Torsional reinforcement

\[ T_n = \frac{T_u}{\phi} = \frac{32.5}{0.75} = 43.33 \text{kN.m} \]

\[ A_t \frac{s}{2A_t f_{yt} \cot \theta} = \frac{43.33 \times 10^6}{2 \times 117983 \times 420 \times 1.0} = 0.437 \text{ mm}^2/\text{mm}/1 \text{ leg} \]

\[ \frac{A_v}{s} = 0.368 \text{ mm}^2/\text{mm}/2 \text{ legs} \quad \text{(From Example 2)} \]

\[ \frac{A_v + A_t}{s} = \frac{2A_t}{s} + \frac{A_v}{s} = 2 \times 0.437 + 0.368 = 1.242 \text{ mm}^2/\text{mm}/2 \text{ legs} \]

Using \( \phi 12 \text{ mm closed stirrups: Area of two legs } = 2 \times 113 = 226 \text{ mm}^2 \)

\[ s = 1.242 \quad \text{Thus } s = \frac{226}{1.242} = 182 \text{ mm} \]

\[ \therefore S = 182 \text{ mm} < S_{max} = 199 \text{ mm} \]
Longitudinal torsion reinforcement

\[ A_t = \frac{A_t}{s} \frac{f_{yt}}{f_y} \cot^2 \theta = 0.437 \times 1592 \times 1.0 \times 1.0 = 695 \text{ mm}^2 \]

Check minimum longitudinal reinforcement:

\[ A_{t \min} = 0.42 \sqrt{f'_c} \frac{A_{cp}}{f_y} - \left( \frac{A_t}{s} \right) p_h \frac{f_{yt}}{f_y} \]

\[ = 0.42 \sqrt{28} \times \frac{220500}{420} - 0.437 \times 1592 \times 1.0 = 471 \text{ mm}^2 \text{ (Controls)} \]

\[ A_{t \min} = 0.42 \sqrt{f'_c} \frac{A_{cp}}{f_y} - \left( \frac{0.175b_w}{f_{yt}} \right) p_h \frac{f_{yt}}{f_y} = 934 \text{ mm}^2 \]

\[ \therefore A_t = 695 \text{ mm}^2 > A_{t \min} = 471 \text{ mm}^2 \]

As in Example 2, \( \frac{1}{3} A_t \) goes to the top corners and \( \frac{1}{3} A_t \) goes to the bottom corners to be added to the flexural bars, while the remaining \( \frac{1}{3} A_t \), would thus be distributed at the mid depth of the beam section.

\[ \sum A_s = \frac{A_t}{3} + A_s = \frac{695}{3} + 1500 = 1732 \text{ mm}^2 \]

\[ N = \frac{1732}{314} = 5.5 \]

Thus, Use \( 6 \varnothing 20 \) at the bottom of the beam

\[ \frac{A_t}{3} = \frac{695}{3} = 231 \text{ mm}^2 \]

Thus, Provide \( \varnothing 16 \) at the top corners, \( A_s = 402 \text{ mm}^2 \)

And provide \( \varnothing 16 \) at mid depth of the beam section, \( A_s = 402 \text{ mm}^2 \)

\[ \therefore \text{use } \varnothing 12 \text{ mm closed stirrups at 180 mm} \]
Homeworks:

1- For the I-beam section shown below, calculate the cracking torque and check the section adequacy for shear and torsion. Consider the factored shear force and the factored torsion at the critical section to be $V_u=180$ kN and $T_u=50$ kN.m. Use $d=690$ mm, $f'_c=25$ MPa, and $f_y=420$ MPa.

![I-beam section diagram]

2- A rectangular beam with section width of 300 mm and section depth of 550 mm is subjected to an external factored shear force acts at the critical section with a value of $V_u=140$ kN and an equilibrium factored external torsional moment of $T_u=30$ kN.m. If $f'_c=28$ MPa, and $f_y=420$ MPa, then design for torsion requirements according to ACI 318-14.

3- A rectangular beam with section width of 250 mm and section depth of 500 mm is subjected to an external factored shear force acts at the critical section with a value of $V_u=200$ kN and compatibility factored external torsional moment of $T_u=40$ kN.m. If $f'_c=28$ MPa, and $f_y=420$ MPa, then design for torsion requirements according to ACI 318-14.

4- For the cantilever beam shown below, If $f'_c=28$ MPa, and $f_y=420$ MPa, then:
   (a) Check the adequacy of the section for shear and torsion requirements,
   (b) Calculate the cracking torsional moment,
   (c) Check if torsion design is required, if yes, then find the required stirrups area and the required longitudinal torsion reinforcement.

![Cantilever beam diagram]