Influence Line of Statically Determinate Structure

Influence lines have important application for the design of structures that resist large live load.

The (I.L) can be defined as diagram that represents the variation of a certain function such as (REACTION, SHEAR, AND MOMENT) when unit load moves on structure.

Aims from Drawing I.L

1. TO PREDICT CRITICAL LOCATION FOR LIVE LOADS OR MOVING LOADS THAT GIVES MAXIMUM VALUE FOR A CERTAIN FUNCTION
2. TO CALCULATE THE MAX VALUE FOR A CERTAIN FUNCTION WHEN THE LIVE LOADS AND MOVING LOADS AT ITS CRITICAL LOCATIONS.

Example 1: draw I.L for reactions at A and B, shear and moment at C and D.

Solution
Put unit load at distance x from point A as shown.

\[ \sum M_B = 0 \]

\[ R_A = \frac{1 \times (14-x)}{14} = 1 - \frac{x}{14} \]

\[ \sum F_Y = 0 \]

\[ R_B = 1 - 1 + \frac{x}{14} = \frac{x}{14} \]
To find shear and moment at point $c$, two cases must be done, first put unit load before point $c$ and the other case by putting unit load after point $c$ as illustrated in figures below

**Case 1**

$0 \leq x \leq 4$

$$\sum f y = 0$$

$$V_c = 1 - \frac{X}{14} - 1 = -\frac{X}{14}$$

$$M_c = \left(1 - \frac{X}{14}\right) \times 4 - 1(4 - X) = \frac{5X}{7}$$

**Case 2**

$4 \leq x \leq 14$

$$\sum f y = 0$$

$$V_c = 1 - \frac{X}{14}$$

$$M_c = 10 \times \frac{X}{14} - 1(X - 4) = -\frac{2X}{7} + 4$$

As homework, find shear and moment at point $D$ (do same procedure for point $c$)

Now, I.L diagram can be constructed depending on equations that obtain previously
Note: the previous method that followed to draw influence line is tedious and needs more time and hard work. There is another way to draw influence line for any function easily. This method developed by HEINRICH MULLER in 1886 and known as HEINRICH MULLER principle. This method can be summarized by following steps.

1. Reaction is considered positive if its direction to upward, to draw I.L for any reaction, move this reaction one unit to upward as shown above for reactions at A and B.

2. Positive shear as shown, to draw I.L for shear function, move left piece to downward while right piece to upward. The value of total movement at point must equal 1.0.

3. Positive moment causes compression at top (make beam smile). to draw I.L, rotate left piece counter clockwise while right piece with clockwise (the total value of angle of rotation must be 1.0). this note is illustrated by following figures.
Notes about I.L for statically determinate structures

1. The path of unit load can be divided into pieces. All piece has ends may be (exterior support, free end, interior hinge or cut).

For I.L ($R_A$) : consider 3 segments (AB , BE, EG)

For I.L ($V_{cl}$) : consider 4 segments (AB ,BC, CE, EG)

For I.L ($M_D$) : consider 4 segments (AB , BD, DE,EG)

2. The supports (exterior or interior) are considered fixed point have zero value at all cases except if influence line is required at this point. At this case, the support move upward one unit while others supports stay fix.

3. I.L for statically determinate structures is straight lines. Therefore, any piece can draw if the values at its ends are predicated.

4. One piece cannot break or bend but it always stay straight.

5. Interior hinge has ability to move if the adjacent piece permits that.

6. The distance of two points of shear equals to 1.0. If right point fixed at its location , then left point drops one unit . If left point fixed at its location then right point rises one unit.

7. Two points of moment stay together at all cases. If one, of them cannot rotate then apply moment to another point and the angle of rotation will be 45.

Example (2):- draw I.L for reactions at a , c, and f , shear at b, e,c , f , g , and j, and moment at a,c,f , g and d.

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Maximum Value of Function

Once the I.L for a function has been constructed, it will then possible to position the live load on the beam that will produce the max value of the function. Two types of loadings will now be considered.

A. Concentrated load $P$(KN)

$F_{\text{max}} = P \times \text{max ordinate of I.L}$

B. Uniform load $w$(KN/m)

$F_{\text{max}} = w \times \text{area of I.L}$

Example(3):- find max positive shear force at point a due to a moving single concentrated load of 50 KN, uniform live load of 2 KN/m and a uniform dead load of 5 KN/m.

Solution

$V_{\text{a live}} = 50(0.6) + 2 \left( \frac{0.6 \times 3}{2} \right) + 2 \left( \frac{0.44 \times 2.2}{2} \right) + 5 \left( \frac{-0.6 \times 3}{2} + \frac{0.6 \times 3}{2} + \frac{-0.4 \times 2}{2} + \frac{0.44 \times 2.2}{2} \right) = 33.2 \text{ KN}$
Example (4):- find max. Tensile force in cable (FG) due to the moving load shown.

\[ \sum M_c = 0 \]
\[ 2T - 0.4 \times 6 = 0 \]
\[ T = 1.2 \text{KN} \]

\[ \frac{X}{3-X} = \frac{4}{6} \]
\[ X = 1.2 \]
1.2 \frac{y_1}{6} = 4.2
\implies y_1 = 0.84

1.2 \frac{y_2}{4} = 2.8
\implies y_2 = 0.84

\[ F_G = 10 \times 1.2 + 2 \left( 0.84 \times 3 + \frac{0.36 \times 3}{2} \right) = 18.12 \text{KN} \]
Influence lines for floor girders

Occasionally, floor systems are constructed as shown in figure below. Where it can be seen that floor loads are transmitted from slabs to floor beams, then to side girders, and finally supporting column. An idealized model of this system is shown in plane view shown below. Here the slab is assumed to be a one-way slab and is segmented into simply supported spans resting on the floor beams. Furthermore, the girder is simply supported on the columns.

Steps of drawing influence line of girder - floor beam system

1. Only girder is considered existing.
2. Draw I.L for required function on basis that girder is ordinary beam.
3. Drop floor beams on drawn I.L at location of its existing.
4. Points of floor beams are connected according to piece of stringer or slab. Exterior supports of stringers are fix point have zero value at datum.
5. In case of drawing I.L of a certain function for stringer itself, the drawing will perform on basis stringer is ordinary beams having supports represented by floor beams

Example :- for the girder- floor-beam – stringer system shown in fig.

1. Draw I.L for Ra & M1-1 in girder
2. Draw I.L for Me in stringer.
Example: - draw I.L for $r_0$, $r_1$, $Ra$, $V_{0-a}$, and $V_{a-1}$.
Influence line for trusses

I.L can be constructed for the bar forces in truss members and are important in determining the location of live loads leading to maximum bar forces in truss members as well as for computing the actual values of these maximum bar forces.

Generally, there are three cases in drawing I.L for truss. These can be summarized as follows:

Case 1:- if the force of bar that required to draw I.L to it can be determined from cut and using $\sum M = 0$ about a certain point, then I.L likes I.L of moment about this point with dealing the truss as beam. However, if the location of point is above then the drawing will be same as beam, while if the point is down then the drawing will be inversely to that of beam.

Example: - draw I.L for bar a & b.

The force at bar a can be determined directly by applying moment equation about point o.

Therefore put unit load at point o and determine the force

$$\sum Mo = 0$$

$$F_a = \frac{0.5 \times 16}{6} = 1.33 \ T$$

Same procedure is applied to bar b

$$\sum Mo' = 0$$

$$F_b = \frac{0.25 \times 24}{6} = 1.00 \ C$$
Case 2: if the force at a certain bar can be determined by cut and using $\Sigma F_y = 0$, then I.L can be constructed by following steps

1- Supports are fixed point
2- Analysis is carried out twice. First by putting unit load at first joint on left cut directly. Second by putting load at first joint on right cut directly.
3- Connect two points and points of supports by straight line. The slope of first line equals of slope of third line.

Example:- draw influence line for member BG
Case 3: if cases 1 & 2 can't be applied, then I.L can be constructed by putting unit load at each joint on path of motion of moving load.

Example: draw I.L for member CG for above truss.

The force at member CG will be zero except when moving load at joint C.
Example: Draw I.L for bar forces a & b

1@ A & E  
Fa=Fb=0

1@ B  
Fa=1.2  
Fb=-5/4

1@ c  
Fa=0  
Fb=5/6

1@ D  
Fa=1.2  
Fb=-5/12
Example: draw I.L for reaction By, $V_{DL}$, $V_{DR}$, and $M_f$
1@ c
\( B_y = 0 \)
\( V_{DL} = -1 \)
\( V_{DR} = 0 \)
\( M_F = 0 \)

1@ D
\( B_y = 0.25 \)
\( V_{DL} = 0 \)
\( V_{DR} = -0.25 \)
\( M_F = -1.5 \)

1@ F
\( B_y = 1 \)
\( V_{DL} = V_{DR} = M_F = 0 \)
Maximum influence at a point due to a series of concentrated loads

Once the influence line of a function has been established for a point in a structure, the maximum effect caused by a live concentrated force is determined by multiplying the peak ordinate of the influence line by the magnitude of the force. In some cases, however, several concentrated forces must be placed on the structure. In order to determine the maximum effect in this case, a trial and error procedure can be used.

Example: find maximum positive shear at point c due to the moving locomotive left to right

**Case 1**

\[ V_c = 4.5(0.75) + 18 \times \frac{0.75}{9} \times (9 - 1.5) + 18 \times \frac{0.75}{9} \times (9 - 3) = 23.63 \text{ KN} \]

**Case 2**

\[ V_c = 4.5 \times \frac{-0.25}{3} (3 - 1.5) + 18 \times 0.75 + 18 \times \frac{0.75}{9} \times (9 - 1.5) = 24.19 \text{ KN} \]

**Case 3**

\[ V_c = 4.5 \times \frac{-0.25}{3} (3 - 3) + 18 \times \frac{-0.25}{3} (3 - 1.5) + 18 \times 0.75 = 11.25 \text{ KN} \]

Maximum positive shear at point c equals to 24.19 kN (case 2)
Example: draw the I.L for the force in member IH of the bridge truss. Compute the maximum live force that can be developed in this member due to a truck having the wheels loads shown. Assume the truck can travel in either direction along the center of the deck, so that half its load is transferred to each of the two side trusses.

Apply unit load at point c and taking left section of cut shown

\[ \sum M_c = 0 \]

\[ F_{IH} = \frac{0.6(12)}{6} = 1.2 \text{ comp.} \]

Now, I.L can be drawn and half load must be taken then apply trial and error method
**Case 1**

\[ F_{IH} = 20(-1.2) + 80 \frac{-1.2}{12} (12 - 4.5) = -84 \text{ kN} \]

**Case 2**

\[ F_{IH} = 20 \frac{-1.2}{18} (18 - 4.5) + 80(-1.2) + 80 \frac{-1.2}{12} (12 - 7.5) = -150 \text{ kN} \]

**Case 3**

\[ F_{IH} = 20 \frac{-1.2}{18} (18 - 12) + 80 \frac{-1.2}{18} (18 - 7.5) + 80(-1.2) = -160 \text{ kN} \]

**Case 4**

\[ F_{IH} = 20(-1.2) + 80 \frac{-1.2}{18} (18 - 4.5) + 80 \frac{-1.2}{18} (18 - 12) = -128 \text{ kN} \]

**Case 5**

\[ F_{IH} = 20 \frac{-1.2}{12} (12 - 4.5) + 80(-1.2) + 80 \frac{-1.2}{18} (18 - 7.5) = -167 \text{ kN} \]

**Case 6**

\[ F_{IH} = 80(-1.2) + 80 \frac{-1.2}{12} (12 - 7.5) = -132 \text{ kN} \]

**Critical case is case 5**
Absolute maximum shear and moment

Trial and error method is developed for computing the maximum shear and moment at a specified point in a beam and truss due to a series of concentrated moving loads. A more general problem involves the determination of both the location of the point in the beam and the position of the loading on the beam so that one can obtain the absolute maximum shear and moment caused by the loads. If the beam is cantilevered or simply supported, this problem can be readily solved.

**Shear:** for a cantilevered beam the absolute maximum shear will occur at a point located just next to the fixed support. For simply supported beams the absolute maximum shear will occur just next to one of the supports.

![Shear Diagram](image)

**Moment:** The absolute maximum moment for a cantilevered beam occurs at the same point where absolute maximum shear occurs, although in this case the concentrated loads should be positioned at the far end of the beam, as shown.

![Moment Diagram](image)

For a simply supported beam the critical position of the loads and the associated absolute maximum moment cannot, in general, be determined by inspection. We can, however, determine the position analytically by following the steps:

1. Determine the resultant and its location
2. Put center line of a beam at half distance between the resultant and nearest haviest load.
3. If the nearest load to resultant is light the try two cases
4. If the length of loads is equal or greater than length of beam then remove immoderate load commonly light load.

Example: calculate the absolute maximum bending moment in a simply span of 24 m under the action of the shown locomotive.

![Locomotive Diagram](image)
R(resultant) = 10 + 20 + 20 = 50 kN

Location of resultant from left point

d = \frac{(20 \times 12 + 20 \times 16)}{50} = 11.2 \text{ m}

To find right support moment about left support will be taken

\[ R_{\text{right}} = \frac{50(12 - 0.4)}{24} = 24.166 \text{ kN} \]

\[ M_{\text{max}} = 24.166(11.6) - 20(4) = 200.333 \text{ kN.m} \]

Example: calculate the absolute maximum bending moment in a simply span of 14 m under the action of the shown locomotive
R = 30*3 + 10 = 100

d = \frac{(10*3 + 30*5 + 30*7)}{100} = 3.9m

Here the nearest load to resultant is not heaviest, so that two cases must be depended.

Case 1 (center line of beam between R & 30kN)

By = 100(7-0.55)/14 = 46.07

M = 46.07(6.45) - 30(2) = 237.15 kN.m

Case 2 (center line of beam between R & 10kN)

By = 100(7+0.45)/14 = 53.32 kN

Mn = 53.32(7-(2-0.45)) - 30(2) = 230.6 kN.m

Mm = 53.32(7+0.45) - 30(2) - 30(4) = 216.41 kN.m
Absolute $M = 237.15 \text{kN.m}$ (case 1 governs)

Example: calculate the absolute maximum bending moment in a simply span of 10m under the action of the shown locomotive.

Length of load equals to length of beam therefore it must reduce to become smaller than length of beam. Smallest load of end loads will remove.

$$R = 20 \times 2 + 40 \times 2 = 120 \text{kN}$$

$$d = (40 \times 2 + 40 \times 5 + 20 \times 7) / 120 = 3.5$$

$$R_A = 120(5 - 0.75) / 10 = 51 \text{kN}$$

$$M = 51(2.25 + 2) - 20(2) = 176.75 \text{kN.m}$$